

# Junior Balkan MO 2012

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- 1] Let  $a, b, c$  be positive real numbers such that  $a + b + c = 1$ . Prove that

$$\frac{a}{b} + \frac{a}{c} + \frac{c}{b} + \frac{c}{a} + \frac{b}{c} + \frac{b}{a} + 6 \geq 2\sqrt{2} \left( \sqrt{\frac{1-a}{a}} + \sqrt{\frac{1-b}{b}} + \sqrt{\frac{1-c}{c}} \right).$$

When does equality hold?

- 2] Let the circles  $k_1$  and  $k_2$  intersect at two points  $A$  and  $B$ , and let  $t$  be a common tangent of  $k_1$  and  $k_2$  that touches  $k_1$  and  $k_2$  at  $M$  and  $N$  respectively. If  $t \perp AM$  and  $MN = 2AM$ , evaluate the angle  $NMB$ .
- 3] On a board there are  $n$  nails, each two connected by a rope. Each rope is colored in one of  $n$  given distinct colors. For each three distinct colors, there exist three nails connected with ropes of these three colors. a) Can  $n$  be 6 ? b) Can  $n$  be 7 ?
- 4] Find all positive integers  $x, y, z$  and  $t$  such that  $2^x 3^y + 5^z = 7^t$ .