

# Junior Balkan MO 2009

---

- 1] Let  $ABCDE$  be a convex pentagon such that  $AB + CD = BC + DE$  and  $k$  a circle with center on side  $AE$  that touches the sides  $AB$ ,  $BC$ ,  $CD$  and  $DE$  at points  $P$ ,  $Q$ ,  $R$  and  $S$  (different from vertices of the pentagon) respectively. Prove that lines  $PS$  and  $AE$  are parallel.
- 2] Solve in non-negative integers the equation  $2^a 3^b + 9 = c^2$
- 3] Let  $x, y, z$  be real numbers such that  $0 < x, y, z < 1$  and  $xyz = (1 - x)(1 - y)(1 - z)$ . Show that at least one of the numbers  $(1 - x)y$ ,  $(1 - y)z$ ,  $(1 - z)x$  is greater than or equal to  $\frac{1}{4}$
- 4] Each one of 2009 distinct points in the plane is coloured in blue or red, so that on every blue-centered unit circle there are exactly two red points. Find the greatest possible number of blue points.