English version

1. Solve the equation

$$3^x - 5^y = z^2$$

in positive integers.

2. Let MN be a line parallel to the side BC of triangle ABC, with M on the side AB and N on the side AC. The lines BN and CM meet at point P. The circumcircles of triangles BMP and CNP meet at two distinct points P and Q. Prove that $\angle BAQ = \angle CAP$.

3. A 9×12 rectangle is partitioned into unit squares. The centres of all the unit squares, except for the four corner squares and the eight squares sharing a common side with one of them, are coloured red. Is it possible to label these red centres $C_1,\ C_2,\ \ldots,\ C_{96}$ in such a way that the following two conditions are both fulfilled

(i) the distances $C_1C_2,\ C_2C_3,\ \ldots,\ C_{95}C_{96},\ C_{96}C_1$ are all equal to $\sqrt{13}$,

(ii) the closed broken line $C_1C_2 \ldots C_{96}C_1$ has a centre of symmetry?

4. Denote by S the set of all positive integers. Find all functions $f : S \to S$ such that

$$f\left(f(m)^2 + 2f(n)^2\right) = m^2 + 2n^2, \qquad \text{for all } m,\ n \in S.$$

Time allowed: 4 hours and 30 minutes.
Each problem is worth 10 points.